Comparison of CO$_2$ Critical Flow Model Based on Henry-Fauske Model with Two-phase Flow

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Introduction

(1) S-CO₂ Brayton Cycle

- Advantages
  - High efficiency at relatively low temperature [450~750°C]
  - Compact components [turbomachinery, heat exchanger]
  - Simple configuration
  - Various applications

DOE, USA, http://www.energy.gov/supercritical-co2-tech-team

V. Dostal, M. J. Driscoll, P. Hejzlar, A Supercritical Carbon Dioxide Cycle for Next Generation Nuclear Reactors, MIT-ANP-TR-100, 2004
Overview Of Supercritical CO₂ Power Cycle Development at Sandia National Laboratories, Steven A. Wright, Thomas M. Conboy, and Gary E. Rochau
Introduction

(2) Research Objectives

● Motivation
  – The **unavoidable leakage** in rotating turbo-machinery **should be addressed**.
    • Since the **S-CO₂ power cycle** is a **highly pressurized system**, certain amount of leakage flow is inevitable in the rotating turbo-machinery via seals.
  – **CO₂ leakage flow** from turbo-machinery via seal becomes one of **important issues**.
    • Since not only it **influences** the **cycle efficiency** due to parasitic loss but also it is important for **evaluating** the **system safety** under various operating conditions
  – To **predict** the **leakage flow rate** and calculate the **required total mass** of working fluid in a S-CO₂ power system
  – To **understand** the **full range of critical flow** in rotating turbo-machinery

→ **A numerical model** for estimating the critical flow in a turbo-machinery seal is essential.

● Goal of this study
  – **CO₂ critical flow modeling** with single phase flow (supercritical and gaseous state) and two phase flow
  – **CO₂ critical flow experiment**
    • To verify the real CO₂ flow behavior and validate the CO₂ critical flow model with experimental results
CO₂ Critical Flow Experimental Facility

(1) Designed Experimental Facility

- **Experiment procedure**
  - **Close** the **ball valve** to separate the high and low pressure tanks
  - **Insert** the **nozzle** between high-pressure CO₂ tank and low-pressure CO₂ tank
  - **Fill** the high-pressure tank with CO₂ from a storage tank until the pressure reaches the maximum pressure
  - **Control** the **initial temperature** of high-pressure CO₂ tank to meet the target point
    - Jacket type heater covered the external of high pressure tank
  - **Set** the **target initial conditions** by controlling the heater and the vent valve
  - **Turn off** the heater and **open** the **ball valve** by hydraulic power of compressed air
  - **Measure all temperatures and pressures** in each point every time until the CO₂ reaches equilibrium

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**Table. Design specifications for experimental system**

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>High/Low-pressure tank</td>
<td></td>
</tr>
<tr>
<td>Pressure (MPa)</td>
<td>22</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>200</td>
</tr>
<tr>
<td>Volume (L)</td>
<td>47</td>
</tr>
<tr>
<td>(I.D.: 200mm, H: 1,500mm)</td>
<td></td>
</tr>
<tr>
<td>Pipe connecting two tanks</td>
<td></td>
</tr>
<tr>
<td>I.D. (mm)</td>
<td>57</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>1090</td>
</tr>
<tr>
<td>Heater (Jacket-type)</td>
<td></td>
</tr>
<tr>
<td>Electric capacity (kW)</td>
<td>5</td>
</tr>
<tr>
<td>Valve type</td>
<td>Ball valve</td>
</tr>
</tbody>
</table>

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**Table. Known constant values and uncertainties for calculation**

<table>
<thead>
<tr>
<th></th>
<th>Known value</th>
<th>Uncertainty</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dₙozzle (mm)</td>
<td>0.5</td>
<td>±0.02</td>
<td>P (kPa)</td>
</tr>
<tr>
<td>Dₜank (mm)</td>
<td>200</td>
<td>±0.5</td>
<td>±(0.00025P)</td>
</tr>
<tr>
<td>Hₜank (mm)</td>
<td>1600</td>
<td>±1.2</td>
<td>T (°C)</td>
</tr>
<tr>
<td>ΔTime (sec)</td>
<td>1</td>
<td>±0.03</td>
<td>±(0.15+0.002T)</td>
</tr>
</tbody>
</table>
(1) CO₂ critical flow modeling with thermal-hydraulic system analysis code (MARS code)

- Multi-dimensional Analysis of Reactor Safety (MARS) critical flow model
  - Henry-Fauske Model

\[ G_C^2 = \left( \frac{x_0 v}{n p} + (v - v_d) \right) \left( 1 - x_0 \right) N \left( \frac{ds_{l,eq}}{dP} \frac{x_0 C_{p,v} \left( \frac{1}{n} - \frac{1}{\gamma} \right)}{P \left( s_{v,eq} - s_{l,eq} \right)} \right)_t^{-1} \]

- Case 1

- Case 2

- Case 3
(1) Simple Modeling (Isentropic CO₂ critical flow model)

- Description of Isentropic CO₂ model

\[ G = \rho V_{\text{velocity}} = \text{constant} \] (1)

\[ \frac{P_0}{P_{\text{critical}}} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}} \] (2)

\[ M = \sqrt{\frac{2}{\gamma-1}} \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} \] (3)

\[ G = \frac{P_0}{\sqrt{RT_0}} \sqrt{\gamma M(1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma+1}{\gamma}}} \] (4)

\[ G_{\text{max}} = \frac{P_0}{\sqrt{RT_0}} \sqrt{\gamma \left(1 + \frac{\gamma-1}{2} \right)^{\frac{\gamma+1}{\gamma}} M_{\text{exit}} = 1.0} \] (5)

- Model Validation

- 13.4MPa, 162°C (Using simple nozzle: \( D=1.5 \text{mm} \), \( L=5.0 \text{mm} \))

Isentropic CO₂ critical flow model

- CO₂ in operating condition behaves like an ideal gas.
  (Compressibility factor ≈ 1)
- CO₂ is stagnant in the CO₂ tanks.
- Whether the flow is choked or not depends on the conditions of high pressure CO₂ tank and the back pressure.
- Choking occurs at the nozzle exit.
- The under-expansion of CO₂ at the nozzle exit is neglected.
(1) Summary and Further Works

- **Summary**
  - Transient simulation of critical flow with MARS code for S-CO$_2$ application
    - To validate the *MARS code* with experimental results, experiments of CO$_2$ critical flow with *simple geometry nozzle* were performed.
      - Simulate the CO$_2$ critical flow *behavior reasonably* when *there is no labyrinth seal geometry effect*.
    - Experiments with nozzles that simulate the condition in the *labyrinth seal* were performed to verify the *number of teeth effect* and to validate the MARS code.
    - MARS results showed *very similar temperature trend* with experimental data, but *mass flux trends are noticeably different* as error accumulates with time due to the *pressure difference*.
      - It seems that this *pressure difference* was caused by the *over estimation of the loss coefficients* in the nozzle geometry.
  - Simple modeling with isentropic CO$_2$ critical flow model
    - Validation of *isentropic CO$_2$ critical flow model* with experimental results using a *simple geometry nozzle* was performed.
      - The experimental mass flux *has similar trend* with the result of isentropic CO$_2$ critical flow model in *single-phase flow*.

- **Further works**
  - To observe *gap effect* in the *isentropic CO$_2$ critical flow model* and to compare with experimental results using different diameter simple geometry nozzles.
  - To reflect *tooth effect* of labyrinth seal on the isentropic CO$_2$ critical flow model and to *validate* this model with experimental results using a *labyrinth seal geometry nozzle*.
  - To identify that the *isentropic CO$_2$ critical flow model* can estimate the dynamic behavior for critical flow with *two-phase condition*.
CO₂ Critical Flow Experiment

(1) Designed nozzles

- Geometry information of designed nozzles

Fig. Geometry information of simple nozzle (left) and Internal geometry of labyrinth seal geometry nozzle (right)

Table. Summary of experimental cases

<table>
<thead>
<tr>
<th></th>
<th>Gap effect</th>
<th>Tooth effect</th>
<th>Two-phase flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 1</td>
</tr>
<tr>
<td>( D \ (mm) )</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( L/D \ (-) )</td>
<td>12.7</td>
<td>38</td>
<td>-</td>
</tr>
<tr>
<td>( Pressure \ ratio \ (-) )</td>
<td>105.0</td>
<td>105.0</td>
<td>105.0</td>
</tr>
<tr>
<td>( N \ (-) )</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>( L_{tooth} \ (mm) )</td>
<td>-</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>( L_{cavity} \ (mm) )</td>
<td>-</td>
<td>-</td>
<td>19</td>
</tr>
<tr>
<td>Diameter ratio (-)</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
CO₂ Critical Flow Experiment

(2) Gap Effect

- **The mass flux** of the CO₂ leak experimental results with simple nozzle and the isentropic CO₂ critical flow model **agrees well with each other**.

- As the **nozzle diameter increases**, the **time** required for reaching equilibrium between the two tanks is **reduced**, which is obvious.

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<th>Case 2</th>
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<td>$D$ (mm)</td>
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</tr>
<tr>
<td>Pressure ratio (-)</td>
<td>105.0</td>
<td>105.0</td>
</tr>
</tbody>
</table>
**Hodkinson’s equation**

Hodkinson modified Egli’s approach to provide a semi empirical relation that was based on assumptions of a gas jet’s geometry. He assumed that the fluid jet expands conically from the tip of an upstream tooth at a small angle.

This expansion is divided into two parts

1. The first part of the jet impinges on the downstream tooth and is forced to re-circulate within the cavity, which leads to the kinetic energy dissipation.
2. The other part of the jet flows under the cavity to the next cavity, carrying the kinetic energy, to the next cavity along with it.

\[
G = \alpha \psi \gamma \sqrt{\rho_i P_i}
\]

\[\alpha = \frac{8.52}{s - w + 7.23} \quad \psi = \sqrt{\frac{1 - \left(\frac{P_e}{P_i}\right)^2}{n - \ln\left(\frac{P_e}{P_i}\right)}} \quad \gamma = \sqrt{\frac{1}{1 - \alpha}}\]

Where, \(G\) is mass flux, \(\alpha\) is relative amount of kinetic energy present upstream of tooth, \(\psi\) is expansion coefficient, \(\gamma\) is kinetic energy carry coefficient, \(P_e\) and \(P_i\) is pressure of low- and high-pressure tanks, \(\rho_i\) is density of high-pressure tank, \(s\) is tooth pitch, \(w\) is tooth width, \(n\) is tooth number, and \(c\) is radial clearance.
CO₂ Critical Flow Experiment

(3) Cavity Length Effect

Fig. Mass flux changes with *cavity length* (upper: KAIST, bottom: University of Wisconsin-Madison)

CO₂ Critical Flow Experiment

(4) Number of Tooth Effect

Fig. Mass flux change with different tooth number (upper: KAIST, bottom: University of Wisconsin-Madison)

Fig. Mach number of experimental result

CO₂ Critical Flow Experiment

(5) Leak Test with Two-phase Flow

- To identify that the developed CO₂ critical flow model can estimate the dynamic behavior for critical flow with two-phase condition (Initial conditions: 15.42MPa, 72.19℃ / 0.101MPa, 25.27℃)
- How to calculate the quality by using only pressure and temperature:
  1. Maintain the HP tank in gaseous or supercritical state (x >1)
  2. Calculate the mass difference on each time step by using properties of HP tank
  3. \[ v(t) = (1 - x)v_f(t) + xv_g(t) \rightarrow x = \frac{v_f(t) - v(t)}{v_f(t) - v_g(t)} \]

Fig. Comparison of mass flux between the experimental and numerical results
Fig. Mach number of experimental result
Fig. T-s diagram of two-phase flow experimental result
(6) Numerical Model (MARS code)

- Multi-dimensional Analysis of Reactor Safety (MARS) code
- The hydrodynamic model simulates transient and steady-state flow behavior of thermal-hydraulic systems
  - Big, complicated, single- and two-phase flows
- The one-dimensional conservation equations for mass, energy and momentum of the flow are solved.
- The field equations are numerically solved:
  - A system is divided into “volumes” and “junctions” -> Nodalization
## CO₂ Critical Flow Experiment

(6) Numerical Model (MARS code)

<table>
<thead>
<tr>
<th>Code (Developer)</th>
<th>Flow model for two-phase flow</th>
<th>Numerical method for flow model</th>
<th>3D T/H model</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>RELAP5/MOD3.3 (INEL, USNRC)</td>
<td>Two-fluid model (6 equations)</td>
<td>Semi-implicit</td>
<td>Available in RELAP5-3D</td>
<td>Systematic V&amp;V, User friendly</td>
</tr>
<tr>
<td>RETRAN-3D (EPRI)</td>
<td>Mixture model (5 equations)</td>
<td>Fully-implicit</td>
<td>None</td>
<td>Non-dominant phase is saturated</td>
</tr>
<tr>
<td>TRAC-PF1/MOD2, TRACE (USNRC)</td>
<td>Two-fluid model (6 equations)</td>
<td>Stability enhancing two-step (SETS)</td>
<td>3D Vessel component</td>
<td>TRACE is under development</td>
</tr>
<tr>
<td>CATHARE 2 (CEA)</td>
<td>Two-fluid model (6 equations)</td>
<td>Fully-implicit (1D), Semi-implicit (3D)</td>
<td>3D Vessel component</td>
<td>Mainly used in EU</td>
</tr>
<tr>
<td><strong>MARS 3.1 (KAERI)</strong></td>
<td><strong>Two-fluid model (1D module), and Two-fluid, three-field model (3D Vessel module)</strong></td>
<td>Semi-implicit</td>
<td>COBRA-TF Vessel module &amp; Multi-D component</td>
<td>Based on the RELAP5 and COBRA-TF codes</td>
</tr>
<tr>
<td>CATHARE 3 (CEA, 2010)</td>
<td>Two-fluid, three-field model</td>
<td>Fully-implicit (1D), Semi-implicit (3D)</td>
<td>3D Vessel component</td>
<td>Turbulence, IAT model, Under development</td>
</tr>
</tbody>
</table>
CO$_2$ Critical Flow Experiment

(6) Numerical Model (MARS code)

- Multi-dimensional Analysis of Reactor Safety (MARS) code
- Input nodalization

- 5 pipes, 9 volumes, one valve and one nozzle
- Giving the heat structure with initial given temperature to give the heat inertia effect
- The Henry-Fauske critical flow model was applied at nozzle
CO₂ Critical Flow Experiment

(6) Numerical Model (MARS code)

• MARS critical flow model

❖ Henry-Fauske Model (2 phase model)

➢ Momentum: \[ -AdP = d(m_v u_v + m_l u_l) + dF_w \]  

\[ -dP = Gd[x u_g + (1-x)u_l] \rightarrow G_t^{-1} = -\left[ \frac{d\{x u_g + (1-x)u_l\}}{dP} \right] \]

➢ At critical flow, \[ \left. \frac{dG_c}{dP} \right|_t = 0 \quad \rightarrow \quad G_c^2 = \left[ -\frac{d}{dP}\left\{ \frac{xk + (1-x)}{k}\{1-x\}k\nu_l + x\nu_g \right\} \right]^{-1} \]

\[ \therefore G_c^2 = \left[ \frac{x_0}{nP} + (\nu_v - \nu_l, 0) \right] \left\{ \frac{(1-x_0)N}{(s_v, eq - s_l, eq)} \frac{ds_{l, eq}}{dP} - \frac{x_0 C_{p,v}(1/n - 1/\gamma)}{P(s_v, 0 - s_l, 0)} \right\}^{-1} \]
CO₂ Critical Flow Experiment

(7) Leak Test with Two-phase Flow

- To identify that the MARS code can estimate the dynamic behavior for critical flow with two-phase condition (Initial conditions: 15.5MPa, 120°C / 0.101MPa, 25.27°C)

- How to calculate the quality by using only pressure and temperature:
  1. Maintain the HP tank in gaseous or supercritical state (x >1)
  2. Calculate the mass difference on each time step by using properties of HP tank
  3. \( v(t) = (1 - x)v_f(t) + xv_g(t) \rightarrow x = \frac{v_f(t) - v(t)}{v_f(t) - v_g(t)} \)

Fig. 1 Comparison of mass flux between the experimental and numerical results

Fig. 2 Mach number of experimental results

Fig. 3 T-s diagram of two-phase flow experimental result

Fig. 4 Specific heat ratio variation of S-CO₂ near the critical point
CO₂ Critical Flow Experiment

(8) Additional Experiment Results

- The two-phase critical flow experiment results

\[ < \text{P}_{\text{cr}} = 19 \text{mm}, D = 0.5 \text{mm}, N = 1 > \]

- CO₂ phase of High-Pressure tank
  - Supercritical -> Liquid -> Gaseous

<table>
<thead>
<tr>
<th></th>
<th>P (MPa)</th>
<th>T (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-pressure tank</td>
<td>8.1</td>
<td>34.5</td>
</tr>
<tr>
<td>Low-pressure tank</td>
<td>0.101</td>
<td>20</td>
</tr>
</tbody>
</table>

Table. Experiment initial condition

Supercritical

Critical point

Liquid

Gaseous
CO$_2$ Critical Flow Experiment

(8) Additional Experiment Results

- The two-phase critical flow experiment results

![Graphs showing pressure, mass flux, and temperature over time for high-pressure and low-pressure tanks with L$_{tooth}=19$mm, D=0.5mm, N=1.]

<table>
<thead>
<tr>
<th></th>
<th>P (MPa)</th>
<th>T (°C)</th>
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<td>20</td>
</tr>
</tbody>
</table>

Table. Experiment initial condition
Summary and Discussion

(9) Experiment Results

- T-s and H-s diagram of each tank
(1) Further works

- Intensive experimental analysis of *two-phase critical flow* will be conducted while special attention is given to the turbo-machinery seal design.

- The *correlations* and *models* for CO₂ two-phase flow in *MARS code* should be re-examined.
  - ✓ Since the MARS code especially focus on the steam tables.

- *More experimental data with two-phase condition* will be reported to study the flow characteristics and to provide validation data for the numerical model.
THANK YOU